**GATE PAPER 2014 (Physics-PH)**

**Q.1 – Q.25 carry one mark each.**

1. The unit vector perpendicular to the surface $x^2 + y^2 + z^2 = 3$ at the point (1, 1, 1) is

(a) $\frac{\hat{x}+\hat{y}-\hat{z}}{\sqrt{3}}$ (b) $\frac{\hat{x}-\hat{y}-\hat{z}}{\sqrt{3}}$ (c) $\frac{\hat{x}-\hat{y}+\hat{z}}{\sqrt{3}}$ (d) $\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}$

2. Which one of the following quantities is invariant under Lorentz transformation?
(a) Charge density (b) Charge (c) Current (d) Electric field

3. The number of normal Zeeman splitting components of $^1P \rightarrow {}^1D$ transition is
(a) 3 (b) 4 (c) 8 (d) 9

4. If the half-life of an elementary particle moving with speed 0.9c in the laboratory frame is $5\times10^{-8}$s, then the proper half-life is _____$\times10^{-8}$ s. ($c = 3\times10^8$ m/s)

5. An unpolarized light wave is incident from air on a glass surface at the Brewster angle. The angle between the reflected and the refracted wave is
(a) 0º (b) 45º (c) 90º (d) 120º

6. Two masses m and 3m are attached to the two ends of a massless spring with force constant K. If m = 100g and K = 0.3 N/m, then the natural angular frequency of oscillation is _____ Hz.

7. The electric field of a uniform plane wave propagating in a dielectric, non-conducting medium is given by,

$$\vec{E} = \hat{x}10\cos\left(6\pi\times10^7\ t - 0.4\pi z\right)V/m$$

The phase velocity of the wave is _____$\times10^8$ m/s.

8. The matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is
(a) orthogonal (b) symmetric (c) anti-symmetric (d) unitary

9. The recoil momemtum of an atom is $p_A$ when it emits an infrared photon of wavelength 1500 nm, and it is $p_B$ when it emits a photon of visible wavelength 500 nm. The ratio $\frac{p_A}{p_B}$ is .

(a) 1 : 1 (b) $1:\sqrt{3}$ (c) 1 : 3 (d) 3 : 2

10. For a gas under isothermal conditions, its pressure P varies with volume V as $P \propto V^{-5/3}$. The bulk modulus B is proportional to

(a) $V^{-1/2}$ (b) $V^{-2/3}$ (c) $V^{-3/5}$ (d) $V^{-5/3}$

11. Which one of the following high energy processes is allowed by conservation laws?

(a) $p + \bar{p} \rightarrow \Lambda^0 + \Lambda^0$ (b) $\pi + p \rightarrow \pi^0 + n$

(c) $n \rightarrow p + e^- + \nu_e$ (d) $\mu^+ \rightarrow e^- + \gamma$

12. The length element ds of an arc is given by, $(ds)^2 = 2\left(dx^1\right)^2 + \left(dx^2\right)^2 + \sqrt{3}\, dx^1 dx^2$. The metric tensor $g_{if}$ is

(a) $\begin{pmatrix} 2 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 1 \\ \sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} \end{pmatrix}$ (d) $\begin{pmatrix} 1 & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{3}{2}} & 2 \end{pmatrix}$

13. The ground state and the first excited state wave functions of a one dimensional infinite potential well are $\psi_1$ and $\psi_2$, respectively. When two spin-up electrons are placed in this potential, which one of the following, with $x_1$ and $x_2$ denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system?

(a) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1) - \psi_1(x_2)\psi_2(x_2)\right]$ (b) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1) + \psi_1(x_2)\psi_2(x_2)\right]$ (d) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)\right]$

14. If the vector potential

$$\vec{A} = \alpha x\hat{x} + 2y\hat{y} - 3z\hat{z}$$

satisfies the Coulomb gauge, the value of the constant $\alpha$ is _____

15. At a given temperature, T, the average energy per particle of a non-interacting gas of two-dimensional classical harmoic oscillator is _____ $k_B T$.

16. Which one of the following is a fermion?

(a) $\alpha$ particle (b) ${}_4Be^7$ nucleus (c) hydrogen atom (d) deuteron

17. Which one of the following three-quark states (qqqq), denoted by X, CANNOT be a possible baryon? The corresponding electric charge is indicated in the superscript

(a) $X^{++}$ (b) $X^{+}$ (c) $X^{-}$ (d) $X^{--}$

18. The Hamilton's canonical equations of motion in terms of Poisson Brackets are

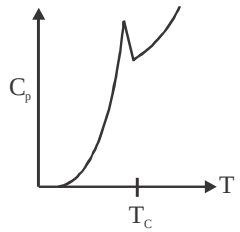
(a) $\dot{q} = \{q, H\};\ \dot{p} = \{p, H\}$ (b) $\dot{q} = \{H, q\};\ \dot{p} = \{H, q\}$

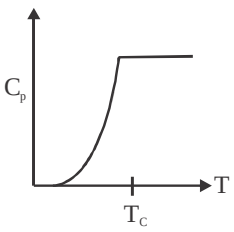
(c) $\dot{q} = \{H, p\};\ \dot{p} = \{H, q\}$ (d) $\dot{q} = \{p, H\};\ \dot{p} = \{q, H\}$

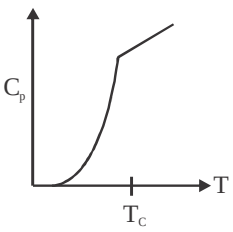
19. The Miller indices of a plane passing through the three points having coordinates (0, 0, 1), (1, 0, 0), $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{4}\right)$ are

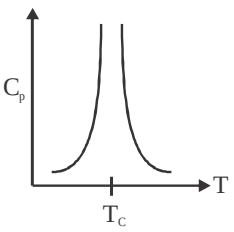
(a) (2 1 2) (b) (1 1 1) (c) (1 2 1) (d) (2 1 1)

20. The plot of specific heat versus temperature across the superconducting transition temperature $(T_C)$ is most appropriately represented by

(a) 


(b) 

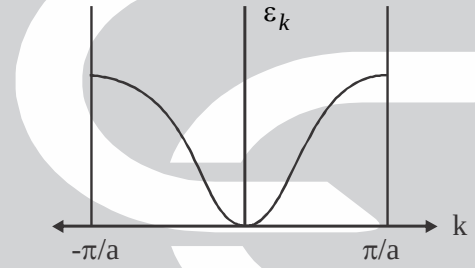

(c) 

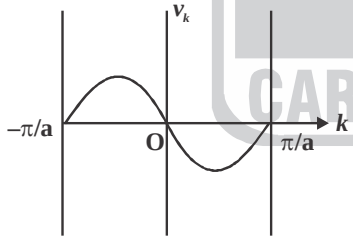

(d) 

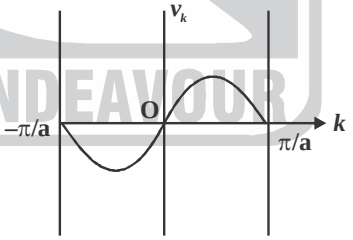

21. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum, then $\vec{L}.\vec{S}$ does NOT commute with

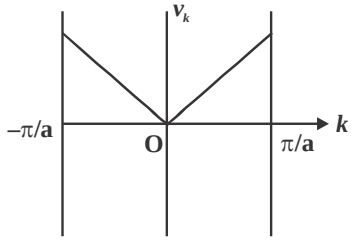
(a) $S_z$ (b) $L^2$ (c) $S^2$ (d) $\left(\vec{L}+\vec{S}\right)^2$

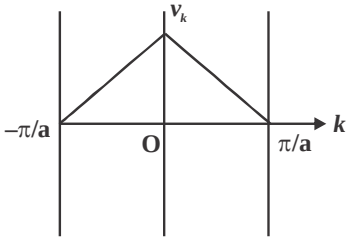
22. The energy, $\varepsilon_k$ for band electrons as a function of the wave vector, k in the first Brillouin zone $\left(-\frac{\pi}{a} \le k \le \frac{\pi}{a}\right)$ of a one dimensional monatomic lattice is shown as (a is latticec constant)



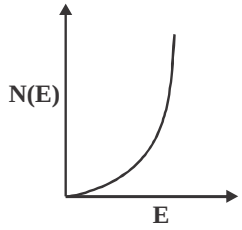

(a) 

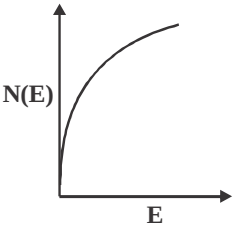

(b) 

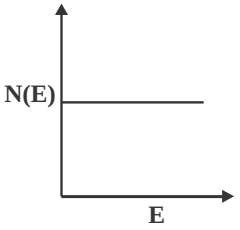

(c) 

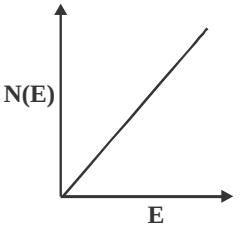

(d) 

23. For a free electron gas in two dimensions, the variation of the density of states, N(E) as a function of energy E, is the best represented by

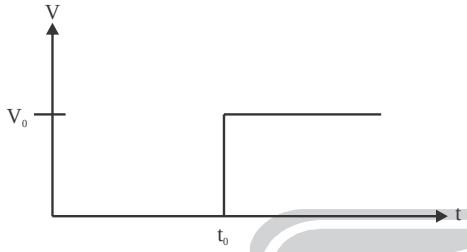
(a) 


(b) 

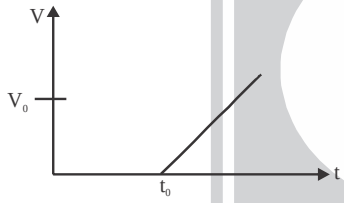

(c) 

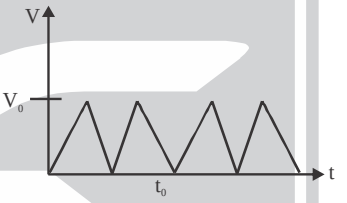

(d) 

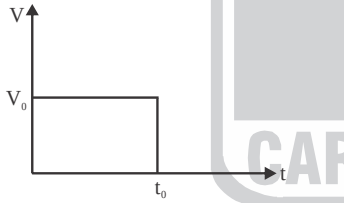

24. The input given to be an ideal OP-AMP integrator circuit is

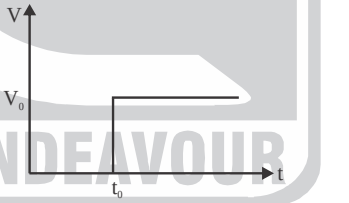



The correct output of the integrator circuit is

(a) 


(b) 


(c) 

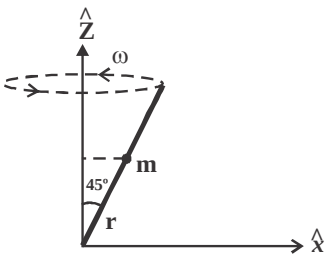

(d) 




25. The minimum number of flip-flops required to construct a mod-75 counter is _____

**Q.26 – Q.55 carry two marks each.**

26. A bead of mass 'm' can slide without friction along a massless rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed $\omega$. At any instant, $r$ is the distance of the bead from the origin. The momentum conjugate to 'r' is




(a) $m\dot{r}$ (b) $\frac{1}{\sqrt{2}}m\dot{r}$ (c) $\frac{1}{2}m\dot{r}$ (d) $\sqrt{2}m\dot{r}$

27. An electron in the ground state of the hydrogen atom has the wave function

$$\Psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-(r/a_0)}$$

where $a_0$ is constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r\cos\theta$ is:

(Hint: $\int_0^\infty e^{-\alpha r} r^n dr = \frac{\Gamma(n)}{\alpha^{n+1}} = \frac{(n-1)!}{\alpha^{n+1}}$)

(a) $-a_0^2/2$ (b) $-a_0^2$ (c) $-3a_0^2/2$ (d) $-2a_0^2$

28. For Nickel, the number density is $8\times10^{23}$ atoms/cm$^3$ and electronic configuration is $1s^2\ 2s^2 2p^6 3s^2 3p^6 3d^8 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic states is _____ $\times 10^9$ A/m.

(Given the value of Bohr magneton $\mu_B = 9.21\times10^{-21}\ Am^2$)

29. A particle of mass 'm' is in a potential given by

$$V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3}$$

where 'a' and '$r_0$' are positive constants. When distrubed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is

(a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (b) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

30. The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300K, assuming the sample to be non degenerate is _____ meV.

$(k_BT = 25meV$ at 300K$)$

31. A particle of mass 'm' is subjected to a potential

$$V(x,y) = \frac{1}{2}m\omega^2\left(x^2+y^2\right), \quad -\infty \le x \le \infty, -\infty \le y \le \infty$$

The state with energy $4\hbar\omega$ is g-fold degenerate. The value of 'g' is _____

32. A hydrogen atom is in the state

$$\Psi = \sqrt{\frac{8}{21}}\psi_{200} - \sqrt{\frac{3}{7}}\psi_{310} + \sqrt{\frac{4}{21}}\psi_{321}$$

where $n, \ell, m$ in $\psi_{n\ell m}$ denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of $\vec{L}$ is _____ $\hbar^2$.

33. A planet of mass 'm' moves in a circular orbit of radius $r_0$ in the gravitational potential $V(r) = -\frac{k}{r}$ where 'k' is a positive constant. The orbital angular momentum of the planet is

(a) $2r_0km$ (b) $\sqrt{2r_0km}$ (c) $r_0km$ (d) $\sqrt{r_0km}$

34. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers $J_A$ and $J_B$ are

(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$ (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

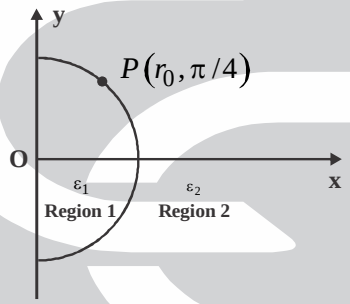
35. The value of the integral

$$\oint_C \frac{z^2}{e^z + 1} dz$$

where C is the circle |z| = 4, is

(a) $2\pi i$ (b) $2\pi^2 i$ (c) $4\pi^3 i$ (d) $4\pi^2 i$

36. A ray of light insid Region 1 in the xy-plane is incident at the semicircle boundary that carries no free charges. The electric field at the point $P(r_0, \pi/4)$ in plane polar coordinates is $\vec{E}_1 = 7\hat{e}_r - 3\hat{e}_\varphi$, where $\hat{e}_r$ and $\hat{e}_\varphi$ are the unit vectors. The emerging ray in Region 2 has the electric field $\vec{E}_2$ parallel to *x*-axis. If $\varepsilon_1$ and $\varepsilon_2$ are the dielectric constants of Region 1 and Region 2 respectively, then $\frac{\varepsilon_2}{\varepsilon_1}$ is _____




37. The solution of the differential equation

$$\frac{d^2y}{dt^2} - y = 0$$

subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$, is

(a) $\cos t + \sin t$ (b) $\cosh t + \sinh t$ (c) $\cos t - \sin t$ (d) $\cosh t - \sinh t$

38. Given that the linear transformation of a generalized coordinate 'q' and the corresponding momentum p,

$$Q = q + 4ap$$
$$p = q + 2p$$

is canonical, the value of the constant 'a' is _____

39. The value of the magnetic field required to maintain non-relativistic protons of energy 1 MeV in a circular orbit of radius 100 mm is _____ Tesla.

(Given: $m_p = 1.67 \times 10^{-27} kg, e = 1.6 \times 10^{-19} C$)

40. For a system of two bosons, each of which can occupy any of the two energy levels 0 and $\varepsilon$, the mean energy of the system at a temperature T with $\beta = \frac{1}{k_B T}$ is given by

(a) $\frac{\varepsilon\, e^{-\beta\varepsilon} + 2\varepsilon\, e^{-2\beta\varepsilon}}{1+2e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$ (b) $\frac{1}{2e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$

(c) $\frac{2\varepsilon\, e^{-\beta\varepsilon} + \varepsilon\, e^{-2\beta\varepsilon}}{2+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$ (d) $\frac{\varepsilon\, e^{-\beta\varepsilon} + 2\varepsilon\, e^{-2\beta\varepsilon}}{2+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$

41. In an interference pattern formed by two coherent sources, the maximum and the minimum of the intensities are $9I_0$ and $I_0$, respectively. The intensities of the individual waves are
(a) $3I_0$ and $I_0$ (b) $4I_0$ and $I_0$ (c) $5I_0$ and $4I_0$ (d) $9I_0$ and $I_0$

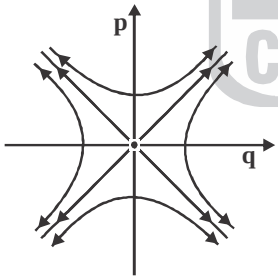
42. $\psi_1$ and $\psi_2$ are two orthogonal states of a spin $\frac{1}{2}$ system. It is given that

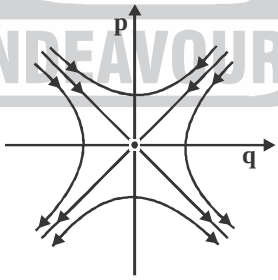
$$\psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}$$

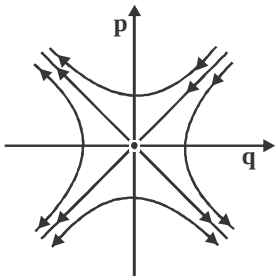
where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represent the spin-up and spin-down states, respectively. When the system is in the state $\psi_2$, its probability to be in spin-up state is _____

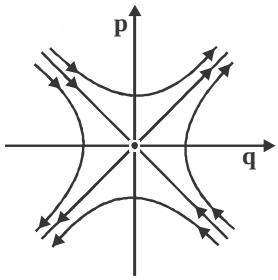
43. Neutrons moving with speed $10^3$ m/s are used for the determination of crystal structure. If the Bragg angle for the first order diffraction is 30°, the interplanarspacing of the crystal is _____Å.
(Given: $m_n = 1.675\times10^{-27}\,kg$, $h = 6.626\times10^{-34}\ J.s$)

44. The Hamiltonian of a particle of mass 'm' is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$. Which of the folloiwng figures describes the motion of the particle in phase space?

(a) 


(b) 


(c) 


(d) 

45. The intensity of a laser in free space is $150\ mW/m^2$. The corresponding amplitude of the electric field of the laser is _____ V/m. $\left(\varepsilon_0 = 8.854\times10^{-12}\ C^2/N.m^2\right)$

46. The emission wavelength for the transition ${}^1D_2 \rightarrow {}^1F_3$ is 3122 Å. The ratio of populations of the final to the initial states at a temperature 5000 K is $(h = 6.626\times10^{-34}\ J.s, c = 3\times10^8\ m/s, k_B = 1.380\times10^{-23}\ J/K)$

(a) $2.03\times10^{-5}$ (b) $4.02\times10^{-5}$ (c) $7.02\times10^{-5}$ (d) $9.83\times10^{-5}$

47. Consider a system of 3 fermions, which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

(a) $k_B \ln 2$ (b) $2k_B \ln 2$ (c) $2k_B \ln 2$ (d) $3k_B \ln 4$

48. A particle is confined to a one dimensional potential box with potential

$$V(x) = 0, \quad 0 < x < a$$
$$= \infty, \quad \text{otherwise}$$

If the particle is subjected to a perturbation, within the box, $W = \beta x$, where $\beta$ is a small constant, the first order correction to the ground state energy is

(a) 0 (b) $a\beta/4$ (c) $a\beta/2$ (d) $a\beta$

49. Consider the process $\mu^+ + \mu^- \rightarrow \pi^+ + \pi^-$. The minimum kinetic energy of the muons (μ) in the centre of mass frame required to produce the pion $(\pi)$ pairs at rest is _____ MeV. (Given : $m_\mu = 105\ MeV/c^2, m_\pi = 140\ MeV/c^2$ )

50. A one dimensional harmonic oscillator is in the superposition of number states, $|n\rangle$, given by

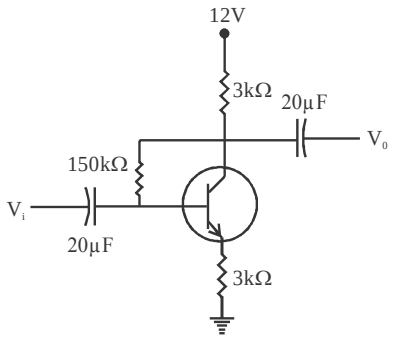
$$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$$

The average energy of the oscillator in the given state is _____ $\hbar\omega$.

51. A nucleus X undergoes a first forbidden $\beta$ -decay to a nucleus Y. If the angular momentum (I) and parity (P), denoted by $I^P$ as $\frac{7^-}{2}$ for X, which of the following is a possible $I^P$ value for Y?

(a) $\frac{1^+}{2}$ (b) $\frac{1^-}{2}$ (c) $\frac{3^+}{2}$ (d) $\frac{3^-}{2}$

52. The current gain of the transistor in the following circuit is $\beta_{dc} = 100$. The value of collector current $I_C$ is _____ mA.




53. In order to measure a maximum of 1V with a resolution of 1mV using a n-bit A/D converter, working under the principle of ladder network, the minimum value of n is _____

54. If $L_+$ and $L_-$ are the angular momentum ladder operators, then, the expectation value of $(L_+L_- + L_-L_+)$, in the state $|\ell = 1, m = 1\rangle$ of an atom is _____ $\hbar^2$

55. A low pass filter is formed by a resistance R and a capacitance C. At the cut-off angular frequency $\omega_c = \frac{1}{RC}$, the voltage gain and the phase of the output voltage relative to the input voltage respectively, are
(a) 0.71 and 45º (b) 0.71 and –45º (c) 0.5 and –90º (d) 0.5 and 90º